

Fundamental Frequencies of U-Tubes in Tube Bundles

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The natural frequencies of U-tubes on multiple supports have been studied as a complement to the author's previous work on the vibration of straight heat-exchanger tubes (reference [1]). A rapid estimation procedure for fundamental frequencies of tubes on symmetrical support spacings has been developed by expressing the frequency in the form

$$f_1 \geq \frac{1}{2\pi} \cdot a_1 \cdot \frac{1}{L_s^2} \cdot \sqrt{\frac{EI}{\mu}}$$

where the square root contains the tube properties of Young's modulus, cross-sectional second moment, and linear density; L_s is a characteristic length of support spacing; and a_1 is a dimensionless number which is a strong function of the support geometry (as expressed by the ratio of bend radius to span lengths) and weak function of Poisson's ratio and of tube axial moment of inertia. a_1 has been plotted as a function of the ratio of the bend radius to the straight-span length, for usual values of Poisson's ratio and small axial moment of inertia. The underlying assumptions for the use of such plots are examined and the theoretical basis for the statement of a lower bound is given, in order to show where the use of this method is applicable.

Nomenclature

a_1 = dimensionless frequency, equals $2\pi f_1 \cdot L_s^2 \cdot$

$$\sqrt{\frac{\mu}{EI}}$$

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- E = Young's modulus of elasticity
- f_1 = lowest natural frequency (Hz)
- G = shear modulus of elasticity
- I = second moment of cross section in bending
- J = polar second moment of cross section
- L_s = unsupported length of straight section (support spacing)
- L_u = unsupported length of U-bend tube centerline
- R = radius of U-bend tube centerline
- μ = mass per unit length of tube
- ν = Poisson's ratio

Introduction

Destructive vibrations in heat exchangers have continued to be a costly and largely unpredictable problem for large segments of American industry (references [2, 3]). The various attempts to model flow-related vibrations (including Y. N. Chen's vortex-shedding model and H. J. Connors whirling model) all depend on the prediction of natural frequencies as the first step [4, 5]. Modeling of resonances induced by pumps or motors also proceeds from the prediction of tube natural frequencies. Similarly, the response to earthquakes depends on natural frequencies. As a result, the prediction of these natural frequencies is an important first step in the design of safe tube bundles.

Straight tubes with arbitrary support spacings, various support lengths and clearances, arbitrary end loads, and fluid-induced inertia in closely packed bundles have been studied experimentally and in computer models, and the results have been represented in tables, graphs, and semi-empirical prediction formulas [1, 6-9]. However, many shell-and-tube and most flue-gas heat exchangers use U-tubes which are more difficult to model. Highly simplified U-tube geometries, with idealized support conditions, have been modeled computationally [10, 11]; a few practical geometries have been tested experimentally [6]. On the basis of the limited available data, "Recommended Design Practices" have been formulated and published with the TEMA Standards [9].

Background

A practical method for determining the lowest natural frequency of a multi-span tube, is to develop a bounded estimate. This approach has been proven successful for straight tubes [1].

To obtain the lower bound of the estimate, the multi-span system is treated as a series of coupled single-span systems. Each single span is represented as a one-degree-of-freedom system having a natural frequency which is no lower than its fundamental frequency as an uncoupled system (i.e., with the tube cut at the intermediate supports). When we now add the coupling of these single-span systems to each other through the slopes and moments of the tube at the intermediate supports, we can reason that any natural frequency of the multi-span system as a whole can be no lower than the lowest uncoupled fundamental frequency determined for any of the single spans, because the effect of the coupling is to add a constraint to the motion of that span.

Since the natural frequencies of single-span straight beams are easily obtained [12] and corrected for axial load [1], immersion in a liquid [8], and noncylindrical tubes [7], the search for the lowest uncoupled fundamental frequency is very rapid. It is only necessary to examine the tubes with the highest axial compressive load, the longest baffle-to-baffle span, and the longest tubesheet-to-baffle span. The lowest frequency value found is the lower bound and a conservative estimate of the first natural frequency of the multi-span system.

To obtain the upper bound of the estimate, we examine the multi-span with the intermediate supports moved to optimal positions for obtaining the highest possible fundamental frequency; it can be reasoned that this is the spacing at which each individual span would have the same uncoupled fundamental frequency—in the simplest case, this means that all the intermediate spans are the same in length and the spans next to the tubesheets 1.25 times as long; in the presence of large axial compressive loads, the end spans need to be up to 1.41 times as long. The fundamental frequency of each uncoupled span and of the multi-span system as a whole is then the same and is an upper bound for the first natural frequency. The upper bound completes the estimate by indicating its range or accuracy. This reasoning also shows that approximately equal support spacings result in the stiffest design for a given number of supports.

The uncoupled natural frequency of the shortest or stiffest span is often used as an upper bound, but is generally not as useful.

Methodology

The procedure of examining uncoupled single spans yields a rapid and generally tightly bounded estimate of lowest natural frequencies of straight tubes in tube bundles. To extend this method to U-tubes, it is necessary to represent the U-bend as a decoupled subsystem in a similar way. For vibrational deflections in the plane of the U-bend, it is still possible to analyze the U-span as an isolated system. However, for the out-of-plane vibrations, cutting off the U-bend leads to a pendulum with a decoupled natural frequency near zero (depending on orientation with respect to gravity), so that no useful information for a lower bound is obtained. In order to obtain useful results, it is necessary to consider a subsystem of two or more spans.

For the symmetrically supported U-tube, we have chosen a three-span subsystem consisting of the U-bend and the adjacent straight span on each side (Fig. 1). In most practical cases, this subsystem has a lower uncoupled natural frequency than the remaining single-span sections, so that its lowest natural frequency is the lower bound of the natural frequency of each entire U-tube.

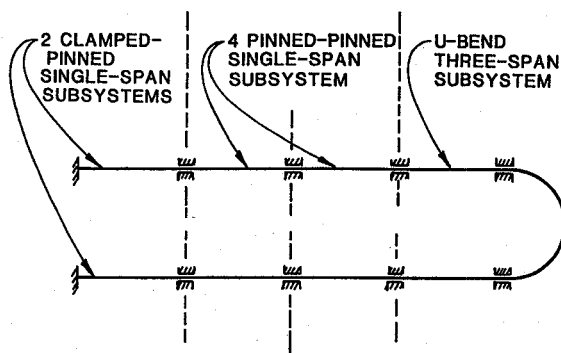


Fig. 1 Division of a U-tube into seven sub-systems for the estimation procedure

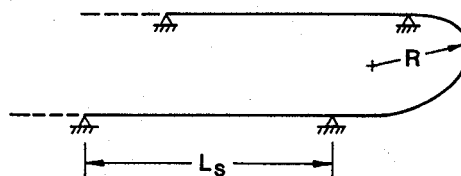


Fig. 2 Elementary three-span U-Tube sub-system geometry

The frequency of this three-span subsystem (Fig. 2) can be expressed in the form

$$f_1 = \frac{1}{2\pi} \cdot a_1 \cdot \frac{1}{L_s^2} \cdot \sqrt{\frac{EI}{\mu}}$$

where the square root contains the tube properties of Young's modulus, cross-sectional second moment, and linear density; L_s is the length of one of the adjacent straight sections; and a_1 is a dimensionless number which is characteristic of the geometry. Expression in this form are familiar for straight spans [12], with a_1 having a constant value for single-span geometry (such as pinned-pinned beam or clamped-pinned beam). For multiple spans, a_1 becomes a function of the ratios of span lengths, such as the ratio of U-bend length to L_s , or of U-bend radius to L_s . Figure 2 shows the simplest (and most commonly applicable) three-span geometry; Fig. 3 shows the corresponding functional dependence of a_1 on the relative size of the U-bend.

In order to obtain a simple curve, three major simplifications were made:

1 The total length of the straight portions of the tube from the U-bend to the tubesheet was assumed to be large, so that torsional restraint of the straight sections could be neglected. This is a conservative assumption.

2 A constant ratio of the bending EI to the torsional GJ was assumed, so that GJ would not appear in the final equation. This can be justified by further assuming that the ratio of the polar second moment J of a circular cross section is twice the bending second moment I (thus neglecting higher-order corrections for the deformation of thin-walled tubes), and that the ratio of the moduli

$$\frac{G}{E} = \frac{1}{2(1+\nu)} \approx 0.39$$

This value is not sensitive to small changes in Poisson's ratio

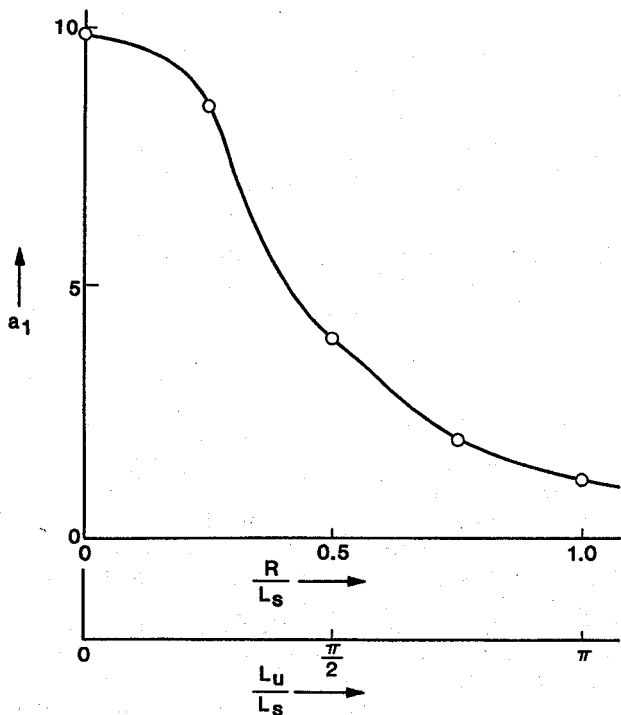


Fig. 3 Dimensionless lowest frequency a_1 of simple symmetrical U-tubes as a function of the geometry ratio R/L_s

ν , and thus is approximately correct for all common metals. Hence

$$\frac{GJ}{EI} \approx 0.78$$

3 The effect of varying rotational inertia of the tube with varying tube sizes was neglected. To include it, we would have to show a whole family of curves for a_1 [10]; for normal tube diameters and span lengths, these curves would be very closely clustered. Hence we have used representative values from shell-and-tube heat exchangers in our computations, and have not shown small variations with scale.

The actual values for the frequency coefficient were obtained in three stages. First, experimental results [6] were studied; they showed the degree to which support conditions can be idealized. Secondly, our present geometry was studied by means of the Rayleigh-Ritz method [11]. This showed us the form of the solution and the influence of the various parameters. Finally, specific values of the frequency were computed numerically by means of the NASTRAN program, leading to the actual plot for a_1 shown in Fig. 3.

Results

For the three-span system shown in Fig. 2, both analysis and computation showed that the first out-of-plane mode of vibration has the lowest natural frequency; depending on the proportions of the dimensions, either the first in-plane mode or the second out-of-plane mode may have the next-lowest

natural frequency. Therefore, the first out-of-plane mode determines a_1 .

For symmetrically supported U-tube, the dimensionless frequency coefficient a_1 for the three-span subsystem shown in Fig. 2 has been computed and plotted in Fig. 3. This value can be used in the estimation procedure outlined in the foregoing. If other straight spans are not significantly longer than L_s , the natural frequency of the entire tube is

$$f_1 \approx \frac{1}{2\pi} \cdot a_1 \cdot \frac{1}{L_s^2} \cdot \sqrt{\frac{EI}{\mu}}$$

In the special case where we are interested only in the in-plane motion of the tube, this figure may be excessively conservative.

Conclusions

The estimation procedure for fundamental frequencies of straight tubes can be extended to U-tubes, and makes rapid results possible. Thus, the benefits of design changes, such as reducing L_s , can be assessed without extensive structural computations.

Application

The purpose of this paper has been to examine the assumptions implied in making simplified plots for estimating the natural frequencies of U-tubes. These assumptions are valid for tubes in ordinary shell-and-tube heat exchangers. Plots for several U-tube geometries have been published in a Technical Note [13] for the use of design engineers.

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