Part I

Simple Systems

Chapter 1

INTRODUCTION and resources

Why do we study vibrations? Because most serious dynamic problems in mechanical engineering involve vibrations. There are two main paths by which vibrations lead to trouble: either a resonance condition permits small disturbances to generate destructively large amplitudes and stresses, or else the frequent repetition of moderate stress reversals leads to premature failure from fatigue or wear. Therefore we must study both resonant and long-enduring oscillations.

1.1 Background

Some case studies in vibrations date back to the nineteenth century, but the number of "trouble jobs" increased markedly with the development of automotive engines in the early twentieth century. As speeds went up, fractured crankshafts became common; machine designers' naïve attempts to "beef up" the shafts, making them heavier, led to even more rapid failures. Solutions were not found until dynamic analysis was applied. Similar problems in big steam turbines were investigated in industry laboratories like the Westinghouse Research Center, and a vibrations discipline was defined by S. Timoshenko and J.P. Den Hartog, who assembled course textbooks in 1928 and 1934, respectively. Since then, the field has gradually expanded as instruments, computer methods, and the mathematical theory of random signals have developed; this book will expand it a little more. We now include the prevention of vibration through suppression, isolation, or balancing; the constructive use of vibrations in technical processes ranging from conveying materials to compacting them; the sources of noise; earthquake and shock loading of structures; airplane wing flutter and other self-sustaining vibrations; cyclic failure; vibration detection and control; non-linear systems; and chaos.

1.2 Objectives

The foundation of vibration analysis is mathematical modeling. At its base is the theory of linear ordinary differential equations. At the next level are the tools of matrix algebra, partial differential equations, non-linear differential equations, and probability and statistics. On top of these rests the study of actual structures, of real earthquakes, of flow-induced vibrations, and so on. The main goal of this book is to establish a solid foundation of physical understanding and mathematical representation, and to apply it to practical trouble-shooting and design.

1.3 Method

Our teaching tools towards this goal include:

- emphasizing fundamentals and de-emphasizing analytical tricks;
- developing physical reasoning hand-in-hand with the mathematics;
- starting with the most elementary theory and methods;
- incorporating realistic and practical problems;
- progressing to advanced topics like damped multi-degree-of-freedom systems;
- introducing modern applications such as experimental modal analysis;
- providing a solid basis for specialized successor courses, such as:
  - acoustics,
  - structural vibrations,
  - numerical methods,
  - instrumentation,
  - signal processing,
  - random vibrations and
  - fatigue,
  - shock loading,
  - earthquake damage,
  - active control,
  - non-linear systems, and
  - chaos;
- using both metric and English units.

1.4 References

This textbook includes the reference information needed to solve the given problems. However, every engineer should have access to a mathematical handbook with which he is familiar. Some possible choices include:

Ronald J. Tallarida, *Pocket Book of Integrals and Mathematical Formulas,* 2nd Edition, CRC Press, Boca Raton, Florida, 1992, ISBN 0-8493-0142-4;

Daniel Zwillinger, editor, *CRC Standard Mathematical Tables and Formulae,* 30th Edition, CRC Press, Boca Raton, Florida, 1996, ISBN 0-8493-2479-3; or

Lennart Råde & B. Westergren, *BETA β Mathematics Handbook: Concepts, Theorems, Methods, Algorithms, Formulas, Graphs, Tables,* 2nd Edition, CRC Press, Boca Raton, Florida, 1992, ISBN 0-8493-7758-7.

1.5 Computers

In order to obtain solutions, calculator and computer procedures are needed. In this book, our emphasis is on putting equations into canonical form so that standard mathematical procedures such as matrix inversion and eigenvalue determination can be used. This will enable the student to utilize computational software like *MATLAB.*[1].

Numerical operations are also available in the symbolic-algebra programs *Maple*[2] and *Mathematica.*[3] Parts of the former are incorporated into the word-processor *Scientific WorkPlace*[4] and the engineering tool *Mathcad.*[5]

Many matrix procedures are possible on scientific calculators.

1.6 Report Writing

Reports on engineering analyses and solutions require a combination of verbal explanations, schematic illustrations[6], and mathematical expressions. This book was developed using *Scientific WorkPlace,*[4] which integrates creation of text, organization of documents using the mark-up-language features of LaTeX, typesetting of mathematical equations using TeX, and mathematical operations and curve-plotting using *Maple.* Additional editing of the resulting LaTeX file was carried out using PC TeX.[7]

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[1] The MathWorks, Inc., Natick, MA 01760-1500, `http://www.mathworks.com/`

[2] Waterloo Maple, Inc., Waterloo, Ontario N2L-5J2, Canada, `http://www.maplesoft.com/`

[3] Wolfram Research, Inc., Champaign, IL 61820-7237, `http://www.wolfram.com/`

[4] MacKichan Software, Bainbridge Island, WA 98110, `http://www.mackichan.com/`

[5] MathSoft, Inc., Cambridge, MA 02142-1521, `http://www.mathsoft.com/`

[6] Edward R. Tufte, *The Visual Display of Quantitative Information,* Second Edition, Graphics Press, Cheshire, Connecticut, © 2001, ISBN 0961392142.

[7] Personal TeX, Inc., Mill Valley, CA 94941, `http://www.pctex.com/`

1.7 Problems

Engineering is learned by solving problems. This introductory set is intended for review of prerequisite material in the area of dynamics. Solve them as a check on your readiness for a vibrations course. If you have trouble with the units, review them in the next chapter.

Problem 1.1 *A meteor has a mass of 6000 slugs (lbf-sec^2/ft). It is moving toward the earth in the equatorial plane, where the earth's gravitational field has an acceleration of 32.09 ft/sec^2 just above the ocean surface. What is the force of gravitational attraction on the meteor just before impact?*

Problem 1.2 *Soon after, a ship of the same mass as the meteor reaches the impact point and stops for observations. What is the force of buoyancy on the ship at this time?*

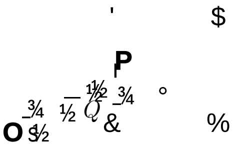
Problem 1.3 *Eventually the ship steams eastward at 40 ft/sec. What is the force of buoyancy during this time?*

Problem 1.4 *A 60-pound boy on skis is pulling on the free end of a rope that is tied to a post. The maximum pull that he can exert without having the rope slip through his grasp is 12 pounds. If the ground is horizontal, calculate the maximum acceleration that he can produce.*

Problem 1.5 *Another skier, who is twice as heavy as the preceding one, unties the rope at the post and proceeds to pull the first boy toward himself. The second boy can exert a maximum pull of 24 pounds. What is the maximum acceleration each skier can experience in this pulling contest?*

Problem 1.6 *A 1750-kg automobile is approaching a crest in an otherwise level road at 50 km/hr. The crest represents a rise of 0.5 m, a length of 8 m, and constant curvature within those eight meters. How much of the car's weight is supported by the road as it passes the crest?*

Problem 1.7 *The sketch represents a circular platform (of radius* $r = 4$ *ft.) rotating about an axis P which itself is mounted on a rotating arm (at a radius of* $R = 8$ *ft.) that rotates about a fixed axis O. The smaller platform rotates with a constant clockwise angular velocity of 7.2 radians/second with respect to the line OP. The longer arm rotates with a constant counterclockwise angular velocity of 3.5 radians/second.*



Calculate the acceleration of the point Q on the edge of the smaller platform, at the moment when Q happens to pass the line OP.

Problem 1.8 *By separation of variables and direct integration, find the solution to the equation*

$$\dot{x} + \alpha x = 0$$

where $\dot{x}$, dx/dt *and the Initial Condition is the displacement* $x_{(0)}$. *Answer: We expect a solution in exponential form.*

Problem 1.9 *Solve the same equation by fitting a power series*

$$x = a_o + a_1 t + a_2 t^2 + a_3 t^3 + ...$$

to the equation and the Initial Condition. Answer: The result should agree with the Maclaurin-series expansion of an exponential function.

Problem 1.10 *Solve the same equation for two positive and two negative values of* α*, using a computer program like* Maple, MATLAB, *or* Mathematica. *Superposition: Since this is a linear equation, you can do this for any initial condition by making the dimensionless ratio* $x/x_{(0)}$ *the dependent variable. Similitude: If we make* (αt) *the dimensionless independent variable, all positive-*α *solutions should be identical, and all negative-*α *solutions should be identical.*

Problem 1.11 *Prepare a comparison of competing software for engineering analysis and simulation. Can you find and discuss alternatives to those already listed in this chapter? Hint: Use the Internet as a resource for commercial information.*

Additional Problems

Problem 1.12 *Identify what type of equation each of these examples is:*

A.

$$\frac{dx}{dt} + x^2 = f(t)$$

B.

$$\frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 x}{\partial y^2} = 0$$

C.

$$\frac{d^2 x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

D.

$$\frac{d^2 x}{dt^2} + \frac{x}{t} = 0$$

Are these equations

- *Ordinary or Partial Differential Equations?*

- *First, second, or third-order?*
- *Linear or non-linear?*
- *with constant or non-constant coefficients?*
- *homogeneous or not?*